① $\int_0^4 |x^2-9|\,dx$

$x^2-9=0$

$(x-3)(x+3)=0$

$x=\pm 3$

$f(2)=(2)^2-9=-5$ ⊖

$f(4)=(4)^2-9=7$ ⊕

$-\int_0^3 x^2-9\,dx + \int_3^4 x^2-9\,dx$

$-\left[\frac{x^3}{3}-9x\right]_0^3 + \left[\frac{x^3}{3}-9x\right]_3^4$

$-\left[\left[\frac{27}{3}-27\right]-0\right] + \left[\left[\frac{64}{3}-36\right]-\left[\frac{27}{3}-27\right]\right]$

$-\left[\frac{-54}{3}\right] + \left[\frac{-44}{3}--\frac{54}{3}\right]$

$\frac{54}{3}+\frac{10}{3}=\boxed{\frac{64}{3}}$

② $f''(x)=4x$, $f'(1)=3$, $f(1)=2$

$f'(x)=\frac{4x^2}{2}+c$

$f'(x)=2x^2+c$

$f'(1)=2(1)^2+c$

$3=2+c$

$c=1$

$f'(x)=2x^2+1$

$f(x)=\frac{2x^3}{3}+x+c$

$2=\frac{2(1)^3}{3}+(1)+c$

$2=\frac{3}{3}+1+c$

$c=\frac{1}{3}$

$f(x)=\frac{2x^3}{3}+x+\frac{1}{3}$

③ $f(x)=3x^2-2$ $[0,3]$ average value $f(c)$

$\frac{1}{3-0}\int_0^3 3x^2-2\,dx \Rightarrow \frac{1}{3}\left[x^3-2x\right]_0^3 \Rightarrow \frac{1}{3}\left(\left[(3)^3-2(3)\right]-\left[(0)^3-2(0)\right]\right)=\frac{1}{3}(21)=\boxed{7}$

④ $\int 2\cos^3 x \sin x\,dx$

$u=\cos x$

$du=-\sin x\,dx$

$\frac{du}{-1}=\frac{-\sin x\,dx}{-1}$

$2\int u^3\left(\frac{du}{-1}\right)=-2\int u^3\,du=-2\cdot\frac{u^4}{4}+c$

$\frac{-\cos^4 x}{2}+c$

⑤ $\frac{dy}{dx}\left[\int_3^{3x^2}(\cos^2 t-3t)\,dt\right]$

2nd Fundamental Th.

$(\cos^2(3x^2)-3(3x^2))6x$

$6x(\cos^2(3x^2)-9x^2)$