

9.4 Rational Expressions

$$1) \frac{4x^2 - 4x - 24}{2x^2 - 6x}$$

$$\frac{4(x^2 - x - 6)}{2x(x-3)}$$

$\begin{matrix} -6 \\ -3 & -2 \end{matrix}$

$$\frac{2\cancel{4}(x-3)(x+2)}{2\cancel{x}(x-3)}$$

$$\boxed{\frac{2(x+2)}{x}}$$

$$2) \frac{3x^2 + 2x - 5}{9x^2 - 25}$$

$$\frac{(3x+5)(x-1)}{(3x+5)(3x-5)}$$

$$\boxed{\frac{x-1}{3x-5}}$$

$$\frac{-15}{5 \times -3}$$

$$(x+5)(x-3)$$

$$(3x)^2 - (5)^2$$

9-4 Rational Expressions

$$1) \frac{12x^5y^{-6}}{4x^{-4}y^2}$$

$$3x^{5-4}y^{-6-2}$$

$$3x^1y^{-8}$$

$$\frac{3x^1}{y^8}$$

$$2) \frac{x^2-16}{5x-20} \quad (x)^2-(4)^2$$

$$\frac{(x-4)(x+4)}{5(x-4)}$$

$$\frac{x+4}{5}$$

$$3) \frac{30x^6}{5y^{-4}} \div \frac{3x^{-2}}{10y}$$

$$\frac{30x^6}{5y^{-4}} \cdot \frac{10y}{3x^{-2}}$$

$$\frac{20x^6y}{x^{-2}y^{-4}}$$

$$20x^8y^5$$

$$3) \frac{8x^2 + 8x}{2-x} \cdot \frac{(x-2)(x+5)}{6x+6}$$

$$\frac{\cancel{4} \cancel{8} x(\cancel{x+1})}{-x+2} \cdot \frac{(\cancel{x-2})(x+5)}{\cancel{3} \cancel{6}(x+1)}$$

~~-(x-2)~~

$$\boxed{\frac{4x(x+5)}{-3}}$$

$$4) \frac{x^2 - 1}{x+4} \div \frac{(x+8)(x-1)^2}{2x+8}$$

$$\frac{(x+1)(\cancel{x-1})}{x+4} \cdot \frac{2(x+4)}{(x+8)(\cancel{x-1})(x-1)}$$

$$\boxed{\frac{2(x+1)}{(x+8)(x-1)}}$$

$$4) \frac{2x^2 - 2}{4x} \div \frac{x^2 + 3x + 2}{2x^2 + 4x}$$

$$\frac{\cancel{1} \cancel{2} (x^2 - 1)}{\cancel{1} \cancel{2} 4x} \cdot \frac{\cancel{2} \cancel{2} (x+2)}{(x+2)(x+1)}$$

$$\frac{x^2 - 1}{x+1} = \frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}}$$

$$(x-1)$$

$$5) \frac{x-1}{x^2 - 5x + 4} \cdot \frac{(12-3x)}{1}$$

$$\frac{\cancel{x-1}}{(\cancel{x-4})(x-1)} \cdot \frac{-3(-4+x)}{1}$$

$$(-3)$$

$$6) \frac{3x^5}{x-5} \cdot \frac{2(x-5)}{6x^3}$$

$$\frac{3x^5}{\cancel{x-5}} \cdot \frac{2(\cancel{x-5})}{6x^3}$$

$$\frac{\cancel{6}x^5}{\cancel{6}x^3} = x^2$$

9-5 Adding and Subtracting Rational Expressions

$$1) \frac{x^2}{4x} + \frac{3x}{4x}$$

$$\frac{x^2+3x}{4x} = \frac{x(x+3)}{4x}$$

$$\frac{x+3}{4}$$

$$2) \frac{x^2+2}{x^2-9} - \frac{11}{x-3} \cdot \frac{(x+3)}{(x+3)}$$

$$\frac{(x-3)(x+3)}{(x^2+2) - (11x+33)}$$

$$\frac{(x-3)(x+3)}{x^2-11x-31}$$

$$\frac{x^2-11x-31}{(x-3)(x+3)}$$

$$3) \frac{(x-1)x}{x^2-x-2} + \frac{(x+5)(x-2)}{x^2-1}$$

$$\frac{(x-1)(x-2)(x+1)}{(x+1)(x-1)(x-2)}$$

$$\frac{(x^2-x) + (x^2+3x-10)}{(x-1)(x-2)(x+1)}$$

$$2x^2+2x-10$$

$$\frac{2x^2+2x-10}{(x-1)(x-2)(x+1)}$$

$$\frac{2(x^2+x-5)}{(x-1)(x-2)(x+1)}$$

9-6 Solving Rational Equations

$$1) \frac{x-3}{x+4} = \frac{2}{x}$$

$$x(x-3) = 2(x+4)$$

$$x^2 - 3x = 2x + 8$$

$$x^2 - 5x - 8 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 + 32}}{2}$$

$$x = \frac{5 \pm \sqrt{57}}{2}$$

$$2) \frac{4x}{4} - x = \frac{3}{2x} \quad \text{LCD: } 4x$$

$$\frac{4x^2}{4} - 4x^2 = \frac{6}{2x}$$

$$x^2 - 4x^2 = 6$$

$$-3x^2 = 6$$

$$\sqrt{x^2} = \sqrt{-2}$$

$$x = \pm i\sqrt{2}$$

$$3) \frac{(x-2)(x+2)}{x^2-4} - \frac{4}{x+2} = \frac{1}{x-2}$$

$$(x-2)(x+2) \text{ LCD}$$

$$\frac{3(x-2)(x+2)}{(x-2)(x+2)} - \frac{4(x-2)(x+2)}{x+2} = \frac{1(x-2)(x+2)}{x-2}$$

$$3 - (4x-8) = x+2$$

$$3 - 4x + 8 = x + 2$$

$$-4x + 11 = x + 2$$

$$-5x + 11 = 2$$

$$-5x = -9 \quad x = \frac{9}{5}$$

Exponent Properties

$$1) a^n \cdot a^m = a^{n+m}$$

$$x^3 \cdot x^5 = x^8$$

$$2) (a^n)^m = a^{n \cdot m}$$

$$(x^3)^4 = x^{12}$$

$$3) \frac{a^n}{a^m} = a^{n-m}$$

$$\frac{x^7}{x^3} = x^4$$

$$4) a^0 = 1$$

$$x^0 = 1$$

$$3^0 = 1$$

$$5) a^{-n} = \frac{1}{a^n}$$

$$x^{-9} = \frac{1}{x^9}$$

$$6) \frac{1}{a^{-n}} = a^n$$

$$\frac{1}{x^{-2}} = x^2$$

$$7) (ab)^n = a^n \cdot b^n$$

$$(3x^4)^2$$

$$3^2 \cdot (x^4)^2 = 9x^8$$

$$8) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\left(\frac{3}{x}\right)^2 = \frac{3^2}{x^2} = \frac{9}{x^2}$$

Factoring

$$1) -12x^3 + 8x^2$$
$$\frac{-4x^2}{\text{GCF}} (\underline{3x} - \underline{2})$$

$$2) 2x^2 + 10x + 12$$
$$\frac{2}{\text{GCF}} (x^2 + \underline{5x} + \underline{6})$$
$$\begin{array}{r} 6 \\ 3 \cancel{ \times } 2 \\ \hline 5 \end{array}$$
$$\underline{2(x+3)(x+2)}$$

$$3) x^2 + 2xy - 8y^2$$
$$\begin{array}{r} -8y^2 \\ 4y \cancel{ \times } -2y \\ \hline 2y \end{array}$$
$$\underline{(x+4y)(x-2y)}$$

$$4) 6x^2 - 5x - 4$$
$$\begin{array}{r} \rightarrow -24 \quad \leftarrow \\ -8 \quad \times \quad 3 \\ \hline -5 \end{array}$$
$$\begin{array}{cc} (x-8) & (x+3) \\ \hline 6 & 6 \end{array}$$
$$\underline{(x-\frac{4}{3})(x+\frac{1}{2})}$$
$$\underline{(3x-4)(2x+1)}$$

$$5) 4x^6 - 81y^2$$
$$(2x^3)^2 - (9y)^2$$
$$\underline{(2x^3-9y)(2x^3+9y)}$$

$$6) 100x^2 - 140x + 49$$
$$(10x)^2 \quad (7)^2$$
$$(10x)(7) = 70x$$
$$\underline{(10x-7)^2}$$

$$7) x^3 + 27$$
$$(x)^3 + (3)^3$$
$$\underline{(x+3)(x^2-3x+9)}$$

$$8) 8x^6 - 125y^3$$
$$(2x^2)^3 - (5y)^3$$
$$\underline{(2x^2-5y)(4x^4+10xy+25y^2)}$$

Fractions

$$1) \frac{\cancel{18} \cdot 10^5}{24 \cdot \cancel{93}} = \frac{30}{36}$$

$$\frac{10^5}{2 \cdot 3} = \left(\frac{5}{6} \right)$$

$$2) \frac{2}{3} \div \frac{12}{5}$$

$$\frac{\cancel{12}}{3} \cdot \frac{5}{\cancel{12} 6} = \left(\frac{5}{18} \right)$$

$$3) \frac{\cancel{3} \cancel{3}}{\cancel{3} 4} - \frac{5}{6} \cdot \frac{2}{2}$$

$$\frac{9}{12} - \frac{10}{12} = \left(\frac{-1}{12} \right)$$

$$\frac{\cancel{4} \cancel{3}}{\cancel{4} 4} + \frac{2}{3} \cdot \frac{4}{4}$$

$$\frac{9}{12} + \frac{8}{12} = \left(\frac{17}{12} \right)$$

$$\left(1 \frac{5}{12} \right) \text{ or}$$