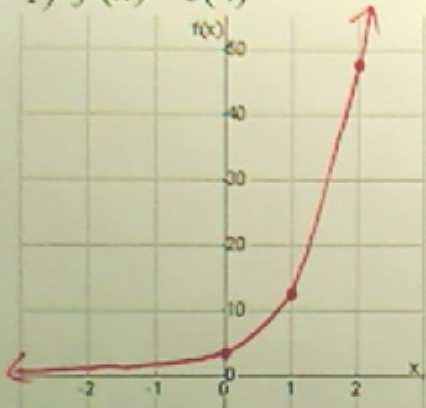
8-1 Exploring Exponential Models

Exponential Function: $y = ab^x$, a = Initial Amount (y-int) b = Growth/Decay Factor (rate)

Exponential Growth: $b > 1$

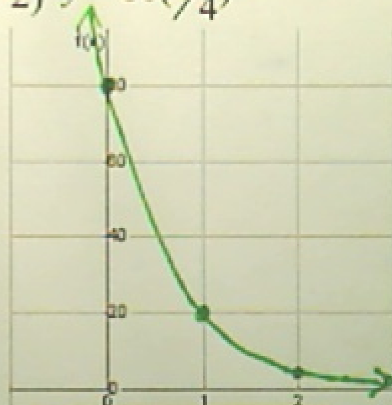
Exponential Decay: $b < 1$ & $b > 0$

1) $f(x) = 3(4)^x$



x	y	*Calculations*
0	3	$3(4)^0 = 3 \cdot 1$
1	12	$3(4)^1 = 3 \cdot 4$
2	48	$3(4)^2 = 3(16)$

2) $y = 80(\frac{1}{4})^x$



x	y	*Calculations*
0	80	$80(\frac{1}{4})^0 = 80(1)$
1	20	$80(\frac{1}{4})^1 = 80(\frac{1}{4})$
2	5	$80(\frac{1}{4})^2 = 80(\frac{1}{16})$

Determine if they are Exponential Growth or Decay:

3) $y = 8(\underline{3.5})^x$

Growth

8 Initial value

4) $y = 0.9^x$

$y = 1(\underline{.9})^x$

Decay

1 Initial value

5) $y = \frac{1}{2}\left(\underline{\frac{5}{4}}\right)^x$

$1\frac{1}{4}$ Growth

6) You own a $10,000 car and it is decreasing in value at about 15% each year. $A = P(1-r)^t$ A=Final Amount P=Principal(Initial Amount) r= rate t=time(yrs)

a) Decay Factor b) Function c) How is it worth in 8 years?

$1-.15$

0.85

$A=10{,}000(1-.15)^t$

$A=10{,}000(.85)^t$

$A=10{,}000(.85)^8$

$2,724.91

7) A soda presently costs $0.50. Inflation increases the cost of things by about 3% each year. $A = P(1+r)^t$

a) Growth Factor b) Function c) How much cost in 20 years?

$1+.03$

1.03

$A=0.50(1.03)^t$

$A=0.5(1.03)^{20}$

$.90

8-3 Logarithmic & Exponential Transformations

1) $\log M = \log_{10} M$	2) $\log_b b = 1$	3) $b^{\log_b x} = x$

1) $9^2 = 81$

$\log_9 81 = 2$

2) $\frac{1}{25} = 5^{-2}$

$5^{-2} = \frac{1}{25}$

$\log_5 \frac{1}{25} = -2$

3) $\log_4 64 = 3$

$4^3 = 64$

4) $\log_{10} 10,000 = 4$

$10^4 = 10,000$

5) $\log_4 16 = x$

$4^x = 16$

$x = 2$

6) $\log_5 \left(\frac{1}{25}\right) = x$

$5^x = \frac{1}{25}$

$x = -2$

7) $\log_{10} 100 = x$

$10^x = 100$

$x = 2$

8-4 Properties of Logarithms

1) $\log_b(MN) = \log_b M + \log_b N$ 2) $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ 3) $\log_b M^N = N \cdot \log_b M$

1) Condense:

a) $\log 7 + 3\log x - 4\log y$

$\log 7 + \log x^3 - \log y^4$

$\log(7x^3) - \log y^4$

$\log\left(\frac{7x^3}{y^4}\right)$

b) $\frac{1}{2}(\log 25 + \log 36) - 8\log x$

$\log 25^{1/2} + \log 36^{1/2} - \log x^8$

$\log 5 + \log 6 - \log x^8$

$\log 30 - \log x^8$

$\log\left(\frac{30}{x^8}\right)$

2) Expand:

a) $\log_4\left(\dfrac{4}{x^2y}\right)$

$\log_4 4 - \log_4 x^2 - \log_4 y$

$1 - 2\log_4 x - \log_4 y$

b) $\log_2 3\sqrt{8a^5}$ $\quad \log_2\left(3(8a^5)^{\frac{1}{2}}\right)$

$\log_2 3 + \log_2(8a^5)^{1/2}$

$\log_2 3 + \frac{1}{2}\log_2(8a^5)$

$\log_2 3 + \frac{1}{2}(\log_2 8 + \log_2 a^5)$

$\log_2 3 + \frac{1}{2}\underbrace{\log_2 8}_{3} + \frac{1}{2}\log_2 a^5$

$\log_2 3 + \frac{3}{2} + \frac{5}{2}\log_2 a$

3) Evaluate: $2\log_2 8 - \log_2 4$

3 2

$\log_2 8 = x$

$2^x = 8$

$x = 3$

$2^x = 4$

$x = 2$

$2(3) - 2 = 4$

$\log_2 8^2 - \log_2 4$

$\log_2 \left(\frac{64}{4}\right)$

$\log_2 16 = x$

$2^x = 16$

4

4) Given:

$\log 5 \approx 0.7$ *and* $\log 6 \approx 0.8$

Find: $\log 150$

$\log(5 \cdot 5 \cdot 6)$

$\log 5 + \log 5 + \log 6$

$.7 + .7 + .8$

$1.4 + .8 = 2.2$

8-5 Exponential and Logarithmic Equations

Change of Base Formula: $\log_b M = \dfrac{\log M}{\log b}$

1) $2(7)^{2x-5} - 6 = 24$

$\quad +6 \quad +6$

$\dfrac{\cancel{2}(7)^{2x-5}}{\cancel{2}} = \dfrac{30}{2}$

$7^{2x-5} = 15$

$\log_7 15 = 2x-5$

$\dfrac{\log 15}{\log 7} \approx 1.4$

$1.4 = 2x-5$

$+5 \quad +5$

$\dfrac{6.4}{2} = \dfrac{2x}{2}$ $x = 3.2$

2) $5^{8x-5} = 25^{3x+1}$

$(5^2)^{3x+1}$ $2(3x+1)$

$5^{8x-5} = 5^{6x+2}$

$8x-5 = 6x+2$

$-6x \quad -6x$

$2x-5 = 2$

$+5 \quad +5$

$\dfrac{\cancel{2}x}{\cancel{2}} = \dfrac{7}{2}$ $x = \dfrac{7}{2}$

3) $2\log_5(3x-2)+1=7$

$-1 \quad -1$

$\frac{\cancel{2}\log_5(3x-2)}{\cancel{2}}=\frac{6}{2}$

$\log_5(3x-2)=3$

$5^3=3x-2$

$125=3x-2$

$+2 \quad +2$

$\frac{127}{3}=\frac{3x}{3}$

$x=\frac{127}{3}$

4) $\log(2x-7)-\log 3=1$

$\log_{10}\left(\frac{2x-7}{3}\right)=1$

$10^1=\frac{2x-7}{3}$

$3\cdot 10=\frac{2x-7}{\cancel{3}}\cdot\cancel{3}$

$30=2x-7$

$+7 \quad +7$

$\frac{37}{2}=\frac{2x}{2}$

$x=\frac{37}{2}$

8-6 Natural Logarithms   e ≈ 2.718

1) $\ln M = \log_e M$	2) $\log_e e = 1$	3) $e^{\log_e x} = x$

1) Condense: $2\ln 5 - \frac{1}{2}\ln x + \ln y$

$\ln 5^2 - \ln x^{\frac{1}{2}} + \ln y$

$\ln 25 - \ln\sqrt{x} + \ln y$

$\ln\left(\frac{25y}{\sqrt{x}}\right)$

2) Simplify: $3 - 4\ln e^5$

$3-4(5\ln e)$ $\log_e e = 1$

$3-4(5\cdot 1)$

$3-4(5)$

$3-20 = -17$

3) $\ln(3n+2)^2 - 4 = 10$

$2\ln(3n+2) - 4 = 10$

$\quad\quad +4 \quad +4$

$\frac{\cancel{2}\ln(3n+2)}{\cancel{2}} = \frac{14}{2}$

$\ln_e(3n+2) = 7$

$e^7 = 3n+2$

$-2 \quad\quad -2$

$\frac{e^7-2}{3} = \frac{\cancel{3}n}{\cancel{3}}$

$n = \frac{e^7-2}{3}$

4) $e^{5x+1} - 2 = 7$

$\quad +2 \quad +2$

$e^{5x+1} = 9$

$\log_e 9 = 5x+1$

$\ln 9 = 5x+1$

$-1 \quad\quad -1$

$\frac{\ln 9 - 1}{5} = \frac{5x}{5}$

$x = \frac{-1+\ln 9}{5}$

5) $\ln(n+5) + \ln(n-5) = 0$

$\ln((n+5)(n-5)) = 0$

$e^0 = (n+5)(n-5)$

$1 = n^2 - 25$

$+25 \quad\quad +25$

$\sqrt{n^2} = \sqrt{26}$

$n = \pm\sqrt{26}$ *

± 5.1

Compounding Continuously: $A = Pe^{rt}$

6) You put \$200 in a CD and it will be worth \$300 in 4 years. What type of interest rate are you getting if interest is compounded continuously?

$A=300 \quad P=200 \quad r=? \quad t=4$

$\frac{300}{200} = \frac{200e^{r \cdot 4}}{200}$

$\frac{3}{2} = e^{4r} \qquad e^{4r} = \frac{3}{2}$

$\log_e(1.5) = 4r$

$\frac{\ln(1.5)}{4} = \frac{4r}{4}$

$r = \frac{\ln(1.5)}{4}$

$r \approx .101$

10%