6.1 Polynomial Functions

1) $(3x^2 - x + 7) - (-7x^2 + 8)$

$(3x^2 - x + 7) + (7x^2 - 8)$

$\boxed{10x^2 - x - 1}$

2) $(3x^3 - x^2 + 7) - (2x^2 - x - 10) + (x - 2)$

$(3x^3 - x^2 + 7) + (-2x^2 + x + 10) + (x - 2)$

$\boxed{3x^3 - 3x^2 + 2x + 15}$

3) $3x^2(-1x^3 + 7x)$

$\boxed{-3x^5 + 21x^3}$

4) $(3x^2 - 7)(2x + 1)$

$\boxed{6x^3 + 3x^2 - 14x - 7}$

5) $(2x - 3)(x^2 + x - 4)$

	$1x^2$	$+x$	-4
$2x$	$2x^3$	$2x^2$	$-8x$
-3	$-3x^2$	$-3x$	12

$\boxed{2x^3 - x^2 - 11x + 12}$

6) $(x-4)^3$ $(x-4)(x-4)(x-4)$

$x^2 - 4x - 4x + 16$

$(x^2 - 8x + 16)(x - 4)$

$(x-4)(x^2 - 8x + 16)$

$x^3 - 8x^2 + 16x - 4x^2 + 32x - 64$

$\boxed{x^3 - 12x^2 + 48x - 64}$

6.3 Dividing Polynomials

1) $(x^2 - 8x + 7) \div (x + 2)$

$$
\begin{array}{r l}
 & x - 10 \\
x+2 \,\big) & x^2 - 8x + 7 \\
(-) & x^2 + 2x \\
\hline
 & -10x + 7 \\
(-) & -10x - 20 \\
\hline
 & 27
\end{array}
$$

$$x - 10 + \frac{27}{x+2}$$

2) $(2x^3 - 5x + 1) \div (x - 4)$

$$
\begin{array}{r l}
 & 2x^2 + 8x + 27 \\
x-4 \,\big) & 2x^3 + 0x^2 - 5x + 1 \\
(-) & 2x^3 - 8x^2 \\
\hline
 & 8x^2 - 5x \\
(-) & 8x^2 - 32x \\
\hline
 & 27x + 1 \\
(-) & 27x - 108 \\
\hline
 & 109
\end{array}
$$

$$2x^2 + 8x + 27 + \frac{109}{x-4}$$

6.4 Solving Polynomial Equations

Difference of Cubes: $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

Sum of Cubes: $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

1) $x^3 - 125$

$(\underset{a}{x})^3 - (\underset{b}{5})^3$

$\boxed{(x-5)(x^2+5x+25)}$

2) $64x^4 + x$

$x(64x^3+1)$

$x((\underset{a}{4x})^3+(\underset{b}{1})^3)$

$\boxed{x(4x+1)(16x^2-4x+1)}$

3) $8x^3 - 27 = 0$

$(\underset{a}{2x})^3 - (\underset{b}{3})^3$

$(2x-3)(4x^2+6x+9)=0$

$2x-3=0$

$+3 \quad +3$

$\frac{2x}{2}=\frac{3}{2}$

$\boxed{x=\frac{3}{2}}$

$4x^2+6x+9=0$

$x=\frac{-6\pm\sqrt{(6)^2-4(4)(9)}}{2(4)}$

$x=\frac{-6\pm\sqrt{36-144}}{8}$

$x=\frac{-6\pm\sqrt{-108}}{8}$

$x=\frac{-6\pm 6i\sqrt{3}}{8}$

$x=\frac{-3\pm 3i\sqrt{3}}{4}$

16
9
144

$\sqrt{-108}$

$i\sqrt{1} \quad \sqrt{108}$

$2\sqrt{4}\cdot\sqrt{27}$

$3\sqrt{9}\sqrt{3}$

6.7 Permutations and Combinations

Factorial: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$

Permutation: (Order) $_nP_r = \dfrac{n!}{(n-r)!}$ $\quad {}_5P_2 = \dfrac{5!}{(5-2)!} = \dfrac{5\cdot4\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 20$

Combination: (Unordered) $_nC_r = \dfrac{n!}{r!(n-r)!} = \dfrac{_nP_r}{r!}$ $\quad {}_5C_2 = \dfrac{5!}{2!(5-2)!} = \dfrac{5\cdot\overset{2}{\cancel{4}}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot1\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = \boxed{10}$

1) $\dfrac{8!}{4!3!}$ $\quad \dfrac{\overset{2}{8}\cdot7\cdot\cancel{6}\cdot5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}$ $\quad 14\cdot20 \quad \boxed{280}$

2) $_7P_5$ $\quad \dfrac{7\cdot6\cdot5\cdot4\cdot3}{42\cdot60}$ $\quad \boxed{2520}$

3) $_6C_{\cancel{4}\,3}$ $\quad \dfrac{_6P_4}{4!} = \dfrac{\cancel{6}\cdot5\cdot\cancel{4}\cdot\cancel{3}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1}$ $\quad 15$

4) $_4P_3 - {}_3C_2$ $\quad 4\cdot3\cdot2 - \dfrac{3\cdot2}{2\cdot1}$ $\quad 24 - 3 = \boxed{21}$

5) Ways to get 1st, 2nd, and 3rd from 6 contestants: $_6P_3 = 6\cdot5\cdot4 = \boxed{120}$

6) Ways to order 3 letter from the word **SUPER:** $_5P_3 = 5\cdot4\cdot3 = \boxed{60}$

7) Ways to choose 5 out of 8 people: $_8C_5 = \dfrac{_8P_5}{5!} = \dfrac{8\cdot7\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}}{\cancel{5}\cdot\cancel{4}\cdot3\cdot2\cdot1} = \boxed{56}$

1) Amigos has 6 meats and 8 fillings to choose from. How many different burritos can you make that have 3 fillings?

$\underset{\text{Meat}}{{}_6C_1} \cdot \underset{\text{Fillings}}{{}_8C_3} = 6 \cdot \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3 \cdot 2} \cdot 1}$

$6 \cdot 56 = \boxed{336 \text{ ways}}$

and ×

or +

2) Pizza Factory has 5 meats and 7 veggies to choose from. How many different pizzas could you make with at most 3 items?

${}_{12}C_3 + {}_{12}C_2 + {}_{12}C_1 + {}_{12}C_0$

$\frac{\overset{2}{\cancel{12}} \cdot 11 \cdot 10}{\cancel{3} \cdot \cancel{2} \cdot 1} + \frac{\overset{6}{\cancel{12}} \cdot 11}{\cancel{2} \cdot 1} + 12 + 1$

$220 + 66 + 12 + 1 = \boxed{299 \text{ ways}}$

6.8 Binomial Theorem

Pascal's Triangle:

$$\begin{array}{lccccccccccc}
 & & & & & & 1 & & & & & \\
 & & & & & 1 & & 1 & & & & \\
{}_2C_r \rightarrow & & & & 1 & & 2 & & 1 & & & \\
{}_3C_r \rightarrow & & & 1 & & 3 & & 3 & & 1 & & \\
{}_4C_r \rightarrow & & 1 & & 4 & & 6 & & 4 & & 1 & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

$${}_4C_2 = \frac{4 \cdot 3}{2 \cdot 1} = 6$$

$${}_4C_3 = \frac{4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 1} = 4$$

$${}_4C_4 = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 1$$

1) $(x^2 - 4y)^5$

$${}_5C_0 (x^2)^5 (-4y)^0 = (1)(x^{10})(1) = x^{10}$$

$${}_5C_1 (x^2)^4 (-4y)^1 = (5)(x^8)(-4y) = -20x^8y$$

$${}_5C_2 (x^2)^3 (-4y)^2 = (10)(x^6)(16y^2) = 160x^6y^2$$

$${}_5C_3 (x^2)^2 (-4y)^3 = (10)(x^4)(-64y^3) = -640x^4y^3$$

$${}_5C_4 (x^2)^1 (-4y)^4 = (5)(x^2)(256y^4) = 1280x^2y^4$$

$${}_5C_5 (x^2)^0 (-4y)^5 = (1)(1)(-1024y^5) = -1024y^5$$

$$\boxed{x^{10} - 20x^8y + 160x^6y^2 - 640x^4y^3 + 1280x^2y^4 - 1024y^5}$$

2) 3rd term of $(x+3)^{8}$

$_8C_0(x)^8(3)^0$

$_8C_1(x)^7(3)^1$

$_8C_2(x)^6(3)^2$

$_8C_2 = \frac{\cancel{8}^4 \cdot 7}{\cancel{2} \cdot 1} = 28$

$3^2 = 9$

$28 \cdot x^6 \cdot 9$

$252x^6$

$$\begin{array}{r} {}^{7}28 \\ 9 \\ \hline 252 \end{array}$$