5.2 Properties of Parabolas

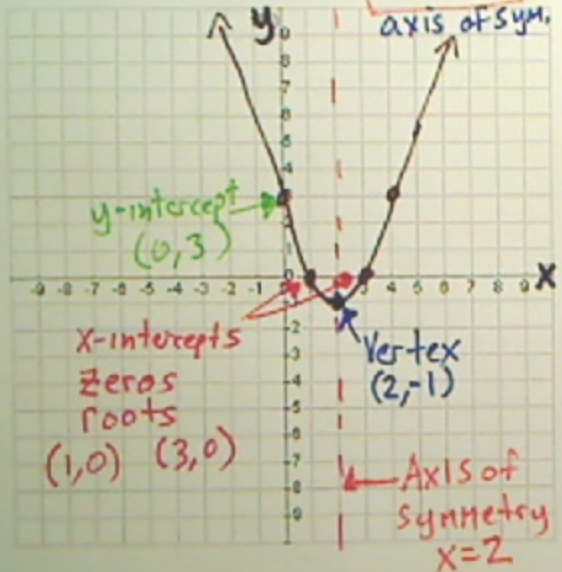
1) Graph: $y = x^2 - 4x + 3$

$x = \frac{-b}{2a}$

$y = ax^2 + bx + c$

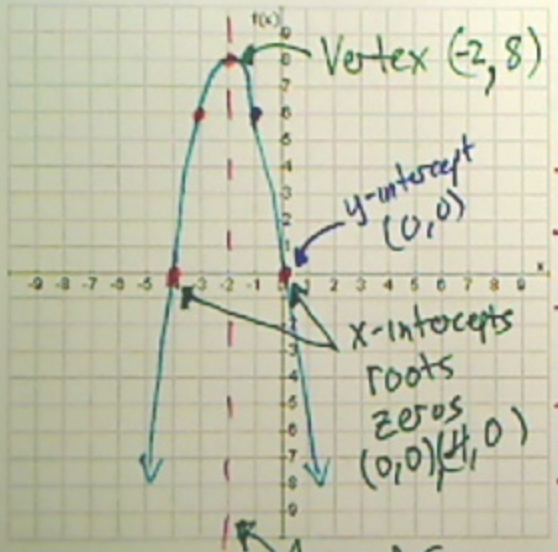
$a=1 \quad b=-4 \quad c=3$

$x = \frac{-(-4)}{2(1)} = \frac{4}{2}$ $x=2$



X	Y	Work
4	3	
3	0	
2	-1	$y=(2)^2-4(2)+3=4-8+3=-1$
1	0	$y=(1)^2-4(1)+3=1-4+3=0$
0	3	$y=(0)^2-4(0)+3=0-0+3=3$

2) Graph: $y = -2x^2 - 8x + 0$ $a = -2$ $b = -8$ $c = 0$



$x = \frac{-b}{2a} = \frac{-(-8)}{2(-2)} = \frac{8}{-4}$ $x = -2$

x	y	
0	0	$-2(0)^2 - 8(0) = 0 - 0 = 0$
-1	6	$-2(-1)^2 - 8(-1) = -2(1) + 8 = 6$
-2	8	$-2(-2)^2 - 8(-2) = -2(4) + 16 = 8$
-3	6	
-4	0	

5.3 Transforming Parabolas

Vertex Form: $y = a(x-h)^2 + k$

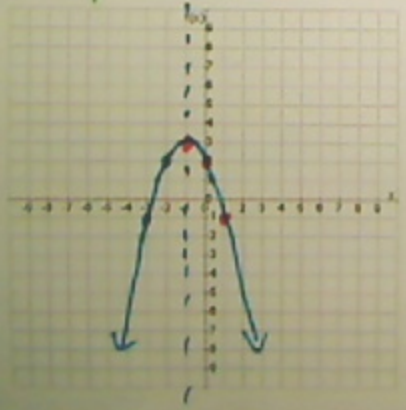
If a is negative: ∩

If a is positive: ∪

1) Graph: $y = -(x+1)^2 + 3$

$(h,k) = (-1,3)$ $a = -1$ ∩ Normal

X	Y	
0	2	$-(0+1)^2+3 = -1+3$
1	-1	$-(1+1)^2+3 = -4+3$



Vertex: (h,k) (x, y)

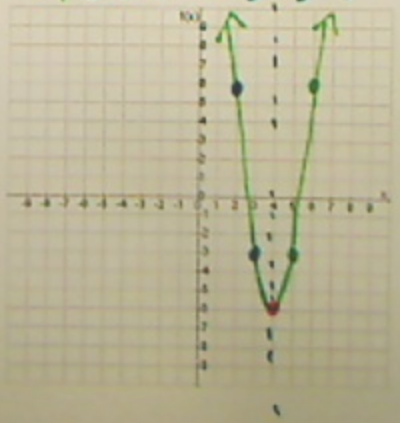
If $|a| < 1$: Wide

If $|a| > 1$: Narrow

2) Graph: $y = 3(x-4)^2 - 6$

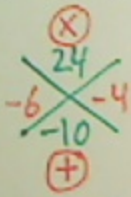
$(h,k) = (4,-6)$ $a = 3$ ∪ Narrow

X	Y	
2	6	$3(2-4)^2-6 = 12-6$
3	-3	$3(3-4)^2-6 = 3-6$



5.4 Factoring Quadratic Expressions

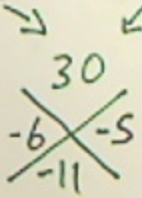
1) $x^2 - 10x + 24$



24 · 1
12 · 2
8 · 3
6 · 4

$(x-6)(x-4)$

2) $2x^2 - 11x + 15$



$(x-\frac{6}{2})(x-\frac{5}{2})$

$(x-3)(2x-5)$

3) $9x^2 - 30x + 25$

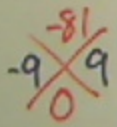
$(3x)^2$ $(5)^2$

$2(3x)(5) = 30x$

$(3x-5)^2$

Perfect Square Trinomial

4) $x^2 - 81$



$(x-9)(x+9)$

$(x)^2 - (9)^2$

Difference of Squares

5) $16x^2 - 49$

$(4x)^2 - (7)^2$

$(4x+7)(4x-7)$

6) $2x^3 - 50x$

$2x(x^2 - 25)$

GCF

$(x)^2 - (5)^2$

$2x(x-5)(x+5)$

7) $3x^2 - 24x - 27$

$3(x^2 - 8x - 9)$

GCF

-9

-9×1

-8

$3(x-9)(x+1)$

5.5 Quadratic Equations

1) $\sqrt{500}$

$\sqrt{100} \cdot \sqrt{5}$

10

$10\sqrt{5}$

2) $\sqrt{32}$

$\sqrt{4}\ \sqrt{8}$

2

$\sqrt{4}\ \sqrt{2}$

2

$2 \cdot 2 \cdot \sqrt{2}$

$4\sqrt{2}$

3) $x^2 - 9x - 36 = 0$

-36, -12, 3, -9

$(x-12)(x+3)=0$

$x-12=0$ $\quad +12\ +12$ $\quad x=12$

$x+3=0$ $\quad -3\ -3$ $\quad x=-3$

4) $x^2 - 11x = 7x - 32$

$-7x\ \ -7x$

$x^2 - 18x = -32$

$+32\ \ +32$

$x^2 - 18x + 32 = 0$

32, -16, -2, -18

$(x-16)(x-2)=0$

$x-16=0$ $\quad x=16$

$x-2=0$ $\quad x=2$

5) $2x^2 - 10x = 4x$

$-4x \quad -4x$

$2x^2 - 14x = 0$

$2x(x - 7) = 0$

$\frac{2x}{2} = \frac{0}{2}$ $\quad x - 7 = 0$

$+7 \quad +7$

$x = 0$ $\quad x = 7$

6) $\frac{4(2x-3)(5x+7)}{4} = \frac{0}{4}$

$(2x-3)(5x+7) = 0$

$2x - 3 = 0$ $\quad 5x + 7 = 0$

$+3 \quad +3$ $\quad -7 \quad -7$

$\frac{2x}{2} = \frac{3}{2}$ $\quad \frac{5x}{5} = \frac{-7}{5}$

$x = \frac{3}{2}$ $\quad x = -\frac{7}{5}$

7) $\frac{9x^2}{9} = \frac{25}{9}$

$\sqrt{x^2} = \sqrt{\frac{25}{9}}$

$x = \pm \frac{\sqrt{25}}{\sqrt{9}}$

$x = \pm \frac{5}{3}$

8) $x^2 - 8 = 12$

$+8 \quad +8$

$\sqrt{x^2} = \sqrt{20}$

$x = \pm\sqrt{20}$

$\sqrt{4}\sqrt{5}$

$x = \pm 2\sqrt{5}$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

5.6 Complex Numbers $(\sqrt{-1})^2$

$i = \sqrt{-1}$ $\quad i^2 = -1$ $\quad i^3 = -\sqrt{-1}$ $\quad i^4 = 1$ $\quad i^5 = \sqrt{-1}$ $\quad i^6 = -1$

1) $\sqrt{-25}$

$\sqrt{-1} \cdot \sqrt{25}$

$i \cdot 5$ $\quad \boxed{5i}$

2) $\sqrt{-50}$

$\sqrt{-1}\ \sqrt{50}$ $\quad \sqrt{50}$

$i\ \sqrt{50}$ $\quad 5 \cdot \sqrt{2}$

$i \cdot 5\sqrt{2}$ $\quad \boxed{5i\sqrt{2}}$

3) $(3-2i)-(-7+4i)$

$(3-2i)+(7-4i)$

$\boxed{10-6i}$

4) $(3-i)(-5+4i)$

$-15+12i+5i-4i^2$

$-15+17i-4(-1)$

$\boxed{-11+17i}$

5) $(-5i)^2$ $\quad (-5i)(-5i)$ $\quad 25i^2$

$25(-1)$

$\boxed{-25}$

6) $\frac{3}{7i} \cdot \frac{7i}{7i} = \frac{21i}{49i^2} = \frac{21i}{49(-1)} = \boxed{-\frac{21i}{49}}$

7) $\frac{3-2i}{2+5i} \cdot \frac{2-5i}{2-5i} = \frac{(3-2i)(2-5i)}{(2+5i)(2-5i)}$ ← conjugate

Top $\quad 6-15i-4i+10i^2$

Bottom $\quad 4-10i+10i-25i^2$

$= \frac{6-19i-10}{4+25}$

$\frac{-4-19i}{29} = \boxed{-\frac{4}{29} - \frac{19}{29}i}$

8) $x^2 + 6 = 2$

$-6 \quad -6$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm\sqrt{-4}$

$\sqrt{-1} \quad \sqrt{4}$

$i \quad 2$

$x = \pm 2i$

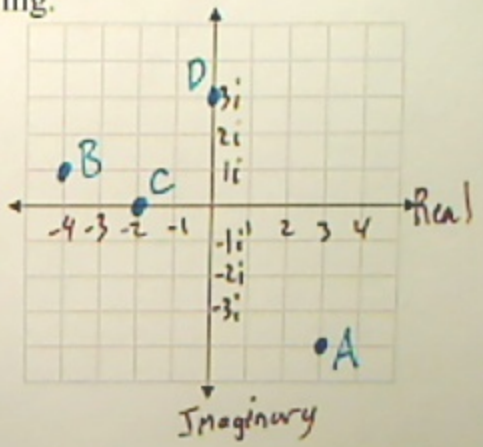
9) Plot the following.

A) $3 - 4i$

B) $-4 + i$

C) $-2 + 0i$

D) $3i + 0$



5.7 Completing the Square

1) $\sqrt{(x+7)^2} = \sqrt{25}$

$x+7 = \pm 5$

$-7 \quad -7$

$x = -7 \pm 5$

$x = -7+5$ $\quad$ $x = -7-5$

$x = -2$ $\quad$ $x = -12$

2) $x^2 - 8x = -20$

$x^2 - 8x + \underline{16} = -20 + \underline{16}$

$\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$16 = -4 \cdot -4 = -8$

$(x-4)(x-4) = -4$

$\sqrt{(x-4)^2} = \sqrt{-4}$

$x - 4 = \pm 2i$

$+4 \quad +4$

$x = 4 \pm 2i$

3) $x^2 + 10x - 2 = 12$

$+2 \quad +2$

$x^2 + 10x + \underline{25} = 14 + \underline{25}$

$\left(\frac{10}{2}\right)^2 = 5^2 = 25$ $\quad$ $25 = 5 \cdot 5 = 10$

$\sqrt{(x+5)^2} = \sqrt{39}$

$x + 5 = \pm\sqrt{39}$

$-5 \quad -5$

$x = -5 \pm \sqrt{39}$

5.8 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Standard Form: $ax^2 + bx + c = 0$

1) $x^2 - 2x - 3 = 0$

$a = 1 \quad b = -2 \quad c = -3$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$x = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$$

$$x = \frac{2+4}{2} = \frac{6}{2} = 3$$

$$x = \frac{2-4}{2} = \frac{-2}{2} = -1$$

2) $2x^2 - 3x = \frac{1}{2}x - 5$

$+5 \quad -x \quad -x \quad +5$

$2x^2 - 4x + 5 = 0$

$a = 2 \quad b = -4 \quad c = 5$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{16 - 40}}{4}$$

$$x = \frac{4 \pm \sqrt{-24}}{4}$$

$\sqrt{-24}$

$\sqrt{-1}\ \sqrt{24}$

$i \quad \sqrt{4}\ \sqrt{6}$

2

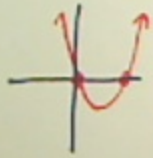
$$x = \frac{4 \pm 2i\sqrt{6}}{4}$$

$$\boxed{x = \frac{2 \pm i\sqrt{6}}{2}} = 1 + \frac{i\sqrt{6}}{2}$$

Discriminant: $b^2 - 4ac$

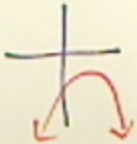
If $b^2 - 4ac > 0$ +

2 Real Solutions



If $b^2 - 4ac < 0$ –

2 Imaginary Solutions
or
∅ Real Solutions



If $b^2 - 4ac = 0$

1 Real Solution

3) $x^2 + 10x - 2 = 0$

$(10)^2 - 4(1)(-2)$

$100 + 8 = 108$

+

Real 2 Solutions

4) $x^2 + 8x + 7 = -9$

$+9 \quad +9$

$x^2 + 8x + 16 = 0$

$(8)^2 - 4(1)(16)$

$64 - 64 = 0$

1 Real Solution

5) $-2x^2 + x - 7 = 0$

$(1)^2 - 4(-2)(-7)$

$8(-7)$

$1 - 56$

-55

0 Real solutions
or
2 Imaginary Solutions