

① Vertical Asymptotes $f(x) = \frac{x^2 - 25}{x^2 + 6x + 5}$

$$f(x) = \frac{(x-5)(x+5)}{(x+5)(x+1)} = \frac{x-5}{x+1} \quad \begin{matrix} x+1=0 \\ x=-1 \end{matrix}$$

② $\lim_{x \rightarrow 3} \frac{3x}{(x-3)^3}$

$\lim_{x \rightarrow 3^-} \frac{3x}{(x-3)^3}$

x	1	2	2.5
y	3	6	∞

$\lim_{x \rightarrow 3^+} \frac{3x}{(x-3)^3}$

x	4	5
y	12	1.875

 ∞ DNE

③ $f(x) = \begin{cases} \frac{x}{x+2}, & x \leq 1 \\ x+3, & x > 1 \end{cases}$ discontinuous

$x+2=0$
 $x=-2$

$f(1) = \frac{1}{1+2} = \frac{1}{3}$

$f(1) = 1+3 = 4$

$x=1$

④ $\lim_{x \rightarrow 0} \frac{(2-2\cos x)}{3x}$

$\lim_{x \rightarrow 0} \frac{2(1-\cos x)}{3x} = \frac{2}{3} \cdot 0 = 0$

⑤ $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$

$\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} \cdot \frac{\sqrt{2+x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{2}} = \lim_{x \rightarrow 0} \frac{(2+x) - 2}{x(\sqrt{2+x} + \sqrt{2})} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{2+x} + \sqrt{2})} = \frac{1}{\sqrt{2+0} + \sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{2}}{4}$

⑥ $\lim_{x \rightarrow -8} \frac{|x+5|}{x+8}$

x	-10	-9	-7	-6
y	$\frac{5}{2}$	4	2	$\frac{1}{2}$

Does not exist