Chapter 4 – Integration

4.1 – Antiderivative and Indefinite Integration

F is the antiderivative of f

Antiderivative = Indefinite Integral

Indefinite Integral: $\int f'(x)dx = f(x) + C$

$$y = f(x) \rightarrow \frac{dy}{dx} = f'(x) \rightarrow dy = f'(x)dx \rightarrow \int dy = \int f'(x)dx \rightarrow y = f(x)$$

Particular Solution – Integrate and then plug in (x,y) to find C.

4.3 – Riemann Sums and Definite Integrals

Riemann Sum: $\sum_{i=1}^{n} f(c_i)\Delta x_i$

If f is continuous and nonnegative on [a,b], then the area of the region bounded by the graph of f, the x-axis, and vertical lines x = a and x = b is given by the definite integral. *(Negative area means it is below the x-axis)*

Definite Integral: $Area = \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx$

Continuity Implies Integrability: If f is continuous on [a,b] then it is integrable on [a,b].

If f is defined at x = a, then $\int_a^a f(x)dx = 0$.

If f is integrable on [a,b], then $\int_b^a f(x)dx = -\int_a^b f(x)dx$.

If f is integrable on the three closed intervals determined by a, b, and c, then $\int_a^b f(x)dx = \int_c^b f(x)dx + \int_a^c f(x)dx$.

4.4 – The Fundamental Theorem of Calculus *Guidelines p.276*

Fundamental Theorem of Calculus: If f is continuous on [a,b] and F is an antiderivative on [a,b], then $\int_a^b f(x)dx = F(b) - F(a)$.

Mean Value Theorem: If f is continuous on [a,b], then there exists a number c in [a,b] such that $\int_a^b f(x)dx = f(c)(b-a)$.

Average Value: If f is integrable on [a,b], then the average value of f on the interval is: $\frac{1}{b-a}\int_a^b f(x)dx$.

Second Fundamental Theorem of Calculus: $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$

4.5 – Integration by Substitution *Guidelines p.292*

U-Substitution (Change of Variables): If $u = g(x)$ *and* $du = g'(x)dx$, then $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$.

If f is an *Even* function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$.

If f is an *Odd* function, then $\int_{-a}^{a} f(x)dx = 0$.

4.6 – Numerical Integration

Trapezoid Rule: $\int_a^b f(x)dx \approx \frac{b-a}{n}\left[f(x_0) + 2f(x_1) + \ldots 2f(x_{n-1}) + f(x_n)\right]$, n = the number of trapezoids.