

## 6.1 Slope Fields

46) Find the general solution:

$$\frac{dy}{dx} = x\sqrt{5-x}$$

$$\int dy = \int x(5-x)^{\frac{1}{2}} dx \quad x=5-u$$

$$y = \int (5-u)u^{\frac{1}{2}} du \quad u=5-x \quad du = -dx$$

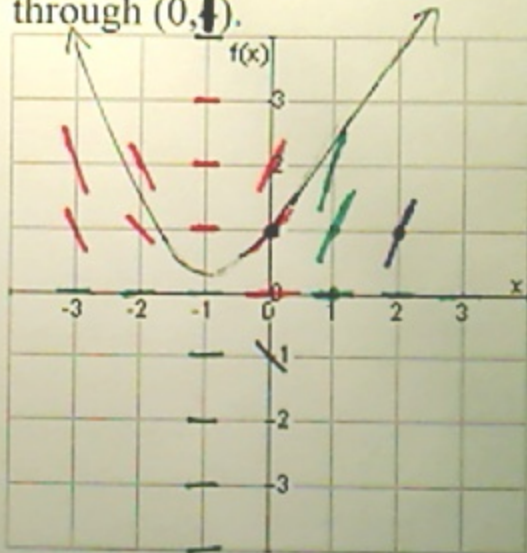
$$y = -1 \int (5u^{\frac{1}{2}} - u^{\frac{3}{2}}) du = dx$$

$$y = -1 \left( \frac{10u^{\frac{3}{2}}}{3} - \frac{2u^{\frac{5}{2}}}{5} \right) + C$$

$$y = -2u^{\frac{3}{2}} \left( \frac{5}{3} - \frac{u}{5} \right) + C$$

$$y = -2(5-x)^{\frac{3}{2}} \left( \frac{5}{3} - \frac{5-x}{5} \right) + C$$

60) Sketch the slope field for  $y' = y + xy'$ . Sketch the solution through  $(0, 1)$ .



$$y' = 0 + 0 \cdot 0 = 0$$

$$y' = 1 + 0(1) = 1$$

$$y' = 2 + 0(2) = 2$$

$$y' = 0 + 1(0) = 0$$

$$y' = 1 + 1(1) = 2$$

$$y' = 2 + 1(2) = 4$$

$$y' = 1 + 2(2) = 5$$

## 6.2 Differential Equations: Growth and Decay $y' = ky \Leftrightarrow y = Ce^{kt}$

10) Solve the differential equation:  $xy + y' = 100x$   
 $-xy \quad -xy$

$$y' = 100x - xy$$

$$y' = x(100 - y)$$

$$\frac{dy}{dx} = x \cdot dx$$

$$\int \frac{1}{100-y} dy = \int x dx$$

$$-\int \frac{1}{u} du = \frac{x^2}{2} + C$$

$$-1 \ln|u| + C = \frac{x^2}{2} + C$$

$$-1 \ln|100-y| = \frac{x^2}{2} + C$$

26) Find the equation  $y = Ce^{kt}$  that passes through  $(0,4)$  and  $(5,1/2)$ .

$$\ln|100-y| = -\frac{x^2}{2} + C$$

$$e^{-\frac{x^2}{2} + C} = |100-y|$$

$$y = -e^{-\frac{x^2}{2} + C} + 100$$

$$y = -e^{-\frac{x^2}{2}} \cdot e^C + 100$$

$$y = -Ce^{-\frac{x^2}{2}} + 100$$

Rate of Change of  $y$ 

$$\hookrightarrow y' = ky \Leftrightarrow y = Ce^{kt}$$

proportional to  $y$

$$A = Pe^{rt}$$

26) Find the equation  $y = Ce^{kt}$  that passes through  $(0, 4)$  and  $(5, 1/2)$ .

$$4 = Ce^{k(0)}$$

Initial Value

 $t = y$ 

$$c = 4$$

$$y = 4e^{kt}$$

$$\frac{1}{2} = \frac{4e^{k(5)}}{4}$$

$$\frac{1}{8} = e^{5k}$$

$$e^{5k} = \frac{1}{8}$$

$$\log_e \frac{1}{8} = 5k$$

$$\frac{\ln(\frac{1}{8})}{5} = \frac{5k}{5}$$

$$y = 4e^{\frac{\ln 8}{-5}t}$$

$$k = \frac{\ln 1 - \ln 8}{5}$$

$$k = \frac{-\ln 8}{5} = \frac{\ln 8}{-5}$$

### 6.3 Separation of Variables

6) Find the general solution:

$$xy' = y$$

$$\frac{y'}{y} = \frac{1}{x} \quad \cancel{dx} \cdot \frac{dy}{dx} = \frac{1}{x} \cdot dx$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| + c = \ln|x| + c$$

$$\ln y = \ln x + c$$

$$\ln y = \ln x + \ln c$$

$$\ln y = \ln(xc)$$

$$y = xc$$

16) Find the particular solution:

$$2xy' - \ln x^2 = 0, \quad y(1) = 2$$

$$y' = \frac{\ln x^2}{2x} \quad dx \cdot \frac{dy}{dx} = \frac{2 \ln x}{2x} \cdot dx$$

$$\int dy = \int \frac{\ln x}{x} dx \quad u = \ln x$$

$$du = \frac{1}{x} dx$$

$$y = \frac{(\ln x)^2}{2} + c$$

$$2 = \frac{(\ln 1)^2}{2} + c$$

$$2 = c$$

$$y = \frac{(\ln x)^2}{2} + 2$$

$$\int u du$$

$$\frac{u^2}{2} + c$$