Chapter 3 - Applications of Differentiation

3.1 – Extrema on an Interval *– Guidelines p.163*

Absolute Extrema - The highest and lowest point on an interval (can include the endpoints).

Relative Extrema - Critical numbers which are a "hill" or "valley".

Critical Numbers - Where f '(c) = 0 or f '(c) does not exist, and where f(c) exists.

3.2 – Rolles's Theorem and the Mean Value Theorem

Rolle's Theorem - If f is continuous [a,b], differentiable (a,b), and f(a) = f(b), then there is at least one number c where f '(c) = 0.

Mean Value Theorem - If f is continuous [a,b] and differentiable (a,b), then there is at least one number c where $f'(c) = \frac{f(b) - f(a)}{b - a}$.

3.3 – Increasing and Decreasing Functions and the First Derivative Test *– Guidelines p.175*

Decreasing/Increasing Functions - If f is continuous [a,b] and differentiable (a,b), then f is **increasing** on [a,b] when f '(c) > 0 on (a,b), f is **decreasing** on [a,b] when f '(c) < 0 on (a,b), and f is **constant** on [a,b] when f '(c) = 0 on (a,b).

First Derivative Test - If c is a critical number (a,b) and differentiable (a,b) except possibly at c, then f(c) is a relative **minimum** when f '(c) changes from – to + , f(c) is a relative **maximum** when f '(c) changes from + to – , and f(c) is **neither** when f '(c) does not change.

3.4 – Concavity and the Second Derivative Test

Concavity - If f "(c) > 0, then concave **upward** (Happy). If f "(c) < 0, then concave **downward** (Sad).

Points of Inflection - A point where the concavity changes, and f "(c) = 0 or does not exist.

Second Derivative Test - If f '(c) = 0 and f "(c) exists, then f(c) is a relative **minimum** when f "(c) > 0 , f(c) is a relative **maximum** when f "(c) < 0 , and you must use the First Derivative Test when f "(c) = 0.

3.5 – Limits at Infinity *– Guidelines p.195*

Limits at Infinity - If r is positive and c is a constant, then $\lim_{x\to\infty} \frac{c}{x^r} = 0$ and $\lim_{x\to-\infty} \frac{c}{x^r} = 0$

Horizontal Asymptotes - If $\lim_{x\to-\infty} f(x) = L$ or $\lim_{x\to\infty} f(x) = L$, then y = L is a horizontal asymptote.

Indeterminate Form - $\lim_{x\to\infty} f(x) = \frac{\infty}{\infty}$ If a limit results in an indeterminate form, then divide both numerator and denominator by the highest degree in the *denominator*.

3.6 – A Summary of Curve Sketching *– Guidelines p.202*

Domain (Possible inputs), Range (Possible outputs), x-intercepts (where y = 0), and y-intercepts (where x = 0)

Vertical Asymptotes - Where a *rational* function produces a non-zero numerator and a denominator of zero.

Slant Asymptotes - Where a *rational* function (having no common factors) has the degree of the numerator exceeding the degree of the denominator by 1. It is the non-rational part of the function after using long division to rewrite a rational function.

Curve Sketching - Find the intercepts, asymptotes, extrema, and points of inflections and then sketch the graph.

3.7 – Optimization Problems *– Guidelines p.212*

Using a **Primary** and a **Secondary Equation,** create an equation with a single independent variable and then find the maximums or minimums of the equation.

3.9 – Differentials

Differentials - If y = f(x), then $\frac{dy}{dx} = f'(x)$. The differential is dy = f '(x)dx. The differential of y (dy) is the change of y (Δy) and the differential of x (dx) is the change of x (Δx).