

$$\textcircled{1} \int \frac{\csc x \cot^3 x}{1 - \csc^2 x} dx$$

$$\textcircled{2} y = \sin x$$

Average $[0, \pi]$

$$\textcircled{3} \int_0^3 2x\sqrt{x+3} dx$$

$$\textcircled{4} F(x) = \int_{x^2}^3 (3t-2) dt, \text{ Find } F'(x).$$

$$\textcircled{1} 1 + \cot^2 x = \csc^2 x$$

$$\cot^2 x = \csc^2 x - 1$$

$$-\cot^2 x = 1 - \csc^2 x$$

$$\int \frac{\csc x \cot^3 x}{-\cot^2 x} dx \rightarrow -\int \csc x \cot x dx = -(-\csc x) + c$$

$$\boxed{\csc x + c}$$

$$\textcircled{2} \frac{1}{\pi} \int_0^{\pi} \sin x dx \rightarrow \frac{1}{\pi} [-\cos x]_0^{\pi} \rightarrow \frac{-1}{\pi} [\cos \pi - \cos 0] \rightarrow \frac{-1}{\pi} [-1 - 1] \rightarrow \frac{-1}{\pi} [-2] = \frac{2}{\pi}$$

$$\textcircled{4} \frac{d}{dx} \left[-\int_3^{x^2} (3t-2) dt \right] = -(3(x^2)-2)(2x) = \boxed{-2x(3x^2-2)} = \boxed{-6x^3 + 4x}$$

$$\textcircled{3} u = x+3$$

$$u = 3+3 = 6$$

$$u = 0+3 = 3$$

$$du = dx$$

$$\int_3^6 2(u-3)u^{1/2} du \Rightarrow \int_3^6 (2u^{3/2} - 3u^{1/2}) du$$

$$x = u - 3$$

$$\rightarrow 2 \left[\frac{2u^{5/2}}{5} - 2u^{3/2} \right]_3^6 \rightarrow 2 \left[\left(\frac{2\sqrt{6^5}}{5} - 2\sqrt{6^3} \right) - \left(\frac{2\sqrt{3^5}}{5} - 2\sqrt{3^3} \right) \right] = \boxed{26.071}$$