

**AP Calculus - Chapter 3 Test Review**

1. Find all critical numbers for the function:  $f(x) = (2x^3 - 2)^{1/4}$
2. Find the absolute maximum and absolute minimum of  $f$  on the interval  $(-1, 1]$ .  $f(x) = \frac{x^3 - x}{1 + x}$
3. Find the absolute extrema of  $f$  on the interval  $[-3, 2]$ .  $f(x) = (x - 1)^{2/3}$
4. Determine why Rolle's Theorem does not apply to the function  $f(x) = \frac{x^3 - 4x}{x}$  on the interval  $[0, 2]$ .
5. Determine why the Mean Value Theorem does not apply to the function  $f(x) = \sqrt{x - 3}$  on the interval  $[2, 4]$ .
6. Given  $f(x) = 2 + \frac{3}{x}$  find all  $c$  in the interval  $(1, 3)$  such that  $f'(c) = \frac{f(3) - f(1)}{3 - 1}$
7. Find all open intervals on which  $f(x) = \frac{x^3 - 2x}{x}$  is increasing.
8. Find all intervals on which the graph of the function is concave upward:  $f(x) = \frac{2x - 1}{x - 2}$
9. Find all intervals on which the graph of the function is concave downward:  $f(x) = \tan x ; \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
10. Find all points of inflection of the graph of the function  $f(x) = x^4 - x^2$ .
11. Let  $f''(x) = x^3 - 2x$  and let  $f(x)$  have critical numbers  $-2, 1,$  and  $2$ . Use the Second Derivative Test to determine which critical numbers, if any, give a relative minimum.
12. Let  $f(x)$  be a polynomial function such that  $f(2) = 3, f'(2) = 5$  and  $f''(2) = 0$ . If  $x < 2$ , then  $f''(2) > 0$  and if  $x > 2$ , then  $f''(2) < 0$ . The point  $(2, 3)$  is a \_\_\_\_\_ of the graph of  $f$ .
13. Let  $f(x)$  be a polynomial function such that  $f(3) = -2, f'(3) = -3$  and  $f''(3) = 0$ . If  $x > 3$ , then  $f''(3) > 0$  and if  $x < 3$ , then  $f''(3) > 0$ . The point  $(3, -2)$  is a \_\_\_\_\_ of the graph of  $f$ .
14. Let  $f(x)$  be a polynomial function such that  $f(2) = 3, f'(2) = 0$  and  $f''(2) = 5$  The point  $(2, 3)$  is a \_\_\_\_\_ of the graph of  $f$ .
15. Let  $f(x)$  be a polynomial function such that  $f(5) = 3, f'(5) = 0$  and  $f''(5) = -5$  The point  $(5, 3)$  is a \_\_\_\_\_ of the graph of  $f$ .
16. Find the slant asymptote for  $f(x) = \frac{x^3 + x}{3 - x^2}$
17. Find the horizontal asymptote for  $f(x) = \frac{5x^2 - 2x}{x^2 + 1}$
18. Find all horizontal asymptotes for  $f(x) = \frac{x + 3}{\sqrt{4x^2 - 5}}$
19. The product of two positive numbers is 180. Maximize the sum of the first and two times the second. What are the two numbers?
20. What is the maximum amount of area you can fence with 400 feet of fencing if you need to make three rectangular pig pens against a barn?
21. An open box is to be made from a 4-foot by 4-foot rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made in this manner.
22. An open box is to be made from a 2-foot by 2-foot rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made in this manner.
23. The side of a cube is measured to be 8.0 inches. If the measurement is correct to within 0.05 inch, use differentials to estimate the propagated error in the volume of the cube.
24. The radius of a sphere is measured to be 6.0 inches. If the measurement is correct to within 0.5 inch, use differentials to estimate the propagated error in the volume of the sphere.
25. Find  $dy$  for  $y = \csc^2 5x$ .

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**Answers**

1.  $x = 1$ , zero is not part of the domain.
2.  $\left(\frac{1}{2}, -\frac{1}{4}\right)$  is an Absolute Minimum and No Absolute Maximum because  $(-1, 2)$  does not exist.
3.  $(1, 0)$  is an Absolute Minimum and  $(-3, \sqrt[3]{16})$  is an Absolute Maximum
4. Not continuous at  $x = 0$  and  $f(0) \neq f(2)$ .
5. Not differentiable from  $[2, 3]$  and not continuous  $[2, 3)$ .
6.  $c = \sqrt{3}$
7.  $(0, \infty)$
8.  $(2, \infty)$
9.  $\left(-\frac{\pi}{2}, 0\right)$
10.  $\left(\pm\frac{\sqrt{6}}{6}, -\frac{5}{36}\right)$
11. 2
12. Point of Inflection
13. Nothing
14. Relative Minimum
15. Relative Maximum
16.  $y = -x$
17.  $y = 5$
18.  $y = \pm\frac{1}{2}$
19. There is no Maximum, the Minimum values would be  $3\sqrt{10}$  &  $6\sqrt{10}$
20. 10,000 ft<sup>2</sup>
21.  $\frac{128}{27} \text{ ft}^3$
22.  $\frac{16}{27} \text{ ft}^3$
23.  $\pm 9.6 \text{ in}^3$
24.  $\pm 72\pi \text{ in}^3$
25.  $dy = -10 \csc^2(5x) \cot(5x) dx$