Expand: $\ln\sqrt{x^3y}$

$\ln(x^3y)^{1/2}$

$\frac{1}{2}\ln(x^3y)$

$\boxed{\frac{1}{2}(3\ln x+\ln y)}$

30) Condense: $3\ln x+2\ln y-4\ln z$

$\ln x^3+\ln y^2-\ln z^4$

$\ln\left(\frac{x^3y^2}{z^4}\right)$

44) Equation of tangent line: $y=\ln x^{1/2}$, $\underset{x\ \ y}{(1,0)}$

$y=\frac{1}{2}\ln x$

$y'=\frac{1}{2}\left(\frac{1}{x}\right)=\frac{1}{2x}$

$m=\frac{1}{2(1)}=\frac{1}{2}$

$y-0=\frac{1}{2}(x-1)$

$y=\frac{1}{2}x-\frac{1}{2}$

58) Derive: $y = \ln \sqrt[3]{\frac{x-1}{x+1}}$ $\quad y = \frac{1}{3}\left(\ln\left(\frac{(x-1)}{(x+1)}\right)\right)$

$$y = \frac{1}{3}\left(\ln(x-1) - \ln(x+1)\right)$$

$$y' = \frac{1}{3}\left(\frac{1}{x-1} - \frac{1}{x+1}\right) = \frac{1}{3}\left(\frac{(x+1)-(x-1)}{(x-1)(x+1)}\right)$$

$$y' = \frac{2}{3(x-1)(x+1)}$$

80) Equation of tangent line: $y^2 + \ln(xy) = 2,\ (e,1)$

$\ln x + \ln y \qquad x \quad y$

$$2y \cdot y' + \frac{1}{x} + \frac{y'}{y} = 0$$

$$2(1)y' + \frac{1}{e} + \frac{y'}{1} = 0$$

$$y - 1 = -\frac{1}{3e}(x - e)$$

$$2y' + y' = -\frac{1}{e} \qquad y' = -\frac{1}{3e}$$

$$3y' = -\frac{1}{e}$$

86) Find relative extrema and inflection points: $y = \dfrac{\ln x}{x}$ $y''(e) = -$

$y' = \dfrac{\frac{1}{x}(x) - (\ln x)(1)}{x^2} = \dfrac{1 - \ln x}{x^2}$ $x^2 = 0$ $x = 0$ DNE

$1 - \ln x = 0$ $\ln x = 1$ $\boxed{x = e}$ $\boxed{\left(e, \tfrac{1}{e}\right) \text{ max}}$

$y'' = \dfrac{\left(-\frac{1}{x}\right)(x^2) - (1 - \ln x)(2x)}{x^4} = \dfrac{-x - 2x(1 - \ln x)}{x^4} = \dfrac{-3x + 2x \ln x}{x^4}$

$y'' = \dfrac{-3 + 2\ln x}{x^3}$ x^4 $-3 + 2\ln x = 0$ $\ln x = \tfrac{3}{2}$ $x = e^{3/2}$ POPI $\left(e^{3/2}, \tfrac{3}{2}e^{-3/2}\right)$

96) Derive using logarithmic differentiation: $y = \sqrt{\dfrac{x^2 - 1}{x^2 + 1}}$

$\ln y = \ln \sqrt{\dfrac{x^2 - 1}{x^2 + 1}} \rightarrow \ln y = \tfrac{1}{2}\left(\ln(x^2 - 1) - \ln(x^2 + 1)\right)$

$\dfrac{y'}{y} = \dfrac{1}{2}\left(\dfrac{2x}{x^2 - 1} - \dfrac{2x}{x^2 + 1}\right) \cdot y$ $y' = \dfrac{2x}{\sqrt{x^2 - 1}\,(x^2 + 1)^{3/2}}$

$y' = \dfrac{2x(x^2 + 1) - 2x(x^2 - 1)}{2(x^2 - 1)(x^2 + 1)} \sqrt{\dfrac{x^2 - 1}{x^2 + 1}} = \dfrac{2x(\cancel{2})}{\cancel{2}(x^2 - 1)(x^2 + 1)} \sqrt{\dfrac{x^2 - 1}{x^2 + 1}}$

14) $\int \frac{2x^2+7x-3}{x-2}\,dx$

$$\begin{array}{r} 2x+11 \\ x-2\,\overline{)\,2x^2+7x-3} \\ (-)\ 2x^2-4x \quad\downarrow \\ \hline 11x-3 \\ (-)\ 11x-22 \\ \hline 19 \end{array}$$

$u=x-2$

$\int 2x+11+\frac{19}{x-2}\,dx \quad du=dx$

$x^2+11x+19\ln|x-2|+c$

28) $\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\,dx$ $\quad u=x^{1/3}-1$

$du=\frac{1}{3}x^{-2/3}\,dx$

$3x^{2/3}\,du=\frac{1}{3x^{2/3}}\,dx\ 3x^{2/3}$

$\int \frac{x^{1/3}}{u}\,du\,(3x^{2/3})$

$3\int \frac{u+1}{u}\cdot(u+1)^2\,du \quad (x^{1/3})^2=(u+1)^2$

$x^{2/3}=(u+1)^2$

$3\int \frac{u^3+3u^2+3u+1}{u}\,du$

$3\int u^2+3u+3+\frac{1}{u}\,du$

$3\left(\frac{u^3}{3}+\frac{3u^2}{2}+3u+\ln|u|\right)+c$

Plug in $x^{1/3}-1$

32) $\int \sec\frac{x}{2}\,dx \quad u=\frac{x}{2} \quad du=\frac{1}{2}dx$

$2\int \sec u\,du$

$2\ln\left|\sec\frac{x}{2}+\tan\frac{x}{2}\right|+c$

44) Particular Solution:

$$\frac{dy}{dx} = \frac{\ln x}{x}, \ (1, -2)$$

$u = \ln x$

$du = \frac{1}{x} dx$

$$\int dy = \int \frac{\ln x}{x} dx$$

$$y = \int u \, du$$

$$y = \frac{u^2}{2} + c$$

$$y = \frac{(\ln x)^2}{2} + c$$

$$-2 = \frac{(\ln 1)^2}{2} + c$$

$c = -2$

$$y = \frac{(\ln x)^2}{2} - 2$$

50) $\displaystyle\int_e^{e^2} \frac{1}{x \ln x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$\ln e^2 = 2 \ln e = \underline{2}$

$\ln e = \underline{1}$

$$\int_1^2 \frac{1}{u} du$$

$$\Big[\ln|u|\Big]_1^2$$

$\ln 2 - \underset{0}{\ln 1}$

$\boxed{\ln 2}$

52) $\int_0^1 \frac{x-1}{x+1}\,dx$ $\quad u = x+1$

$du = dx$

$1+1=2$

$0+1=1$ $\quad x = u-1$

$$\int_1^2 \frac{(u-1)-1}{u}\,du$$

$$\int_1^2 \frac{u-2}{u}\,du$$

$$\int_1^2 1-\frac{2}{u}\,du$$

$$\left[u-2\ln|u|\right]_1^2$$

$$(2-2\ln 2)-(1-\underbrace{2\cancel{\ln 1}}_{0})$$

$$1-\ln 4$$

70) $\int_{\pi/4}^{3\pi/4} \frac{\sin x}{1+\cos x}\,dx$ $\quad u = 1+\cos x$

$du = -\sin x\,dx$

$$\int \frac{-1}{u}\,du = -\ln|u|$$

$$\left[-\ln|1+\cos x|\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$-1\left(\ln\left|1+\left(-\frac{\sqrt{2}}{2}\right)\right| - \ln\left|1+\frac{\sqrt{2}}{2}\right|\right)$$

$$\ln\left|1+\frac{\sqrt{2}}{2}\right| - \ln\left|1-\frac{\sqrt{2}}{2}\right|$$

Solve: a) $2e^{4x} - 5 = 11$

$\quad\;\; +5 \quad +5$

$$\frac{2e^{4x}}{2} = \frac{16}{2}$$

$$e^{4x} = 8$$

$$\log_e 8 = 4x$$

$$\frac{\ln 8}{4} = \frac{4x}{4} \qquad x = \frac{\ln 8}{4}$$

b) $4 - \ln(x+5)^3 = 22$

$-4 \qquad\qquad -4$

$$\frac{-3\ln(x+5)}{-3} = \frac{18}{-3}$$

$$\ln_e(x+5) = -6$$

$\ln(e^{-6})$

$$e^{-6} = x+5$$

$-5 \qquad -5$

$$x = e^{-6} - 5$$

38) Derive: $y = x^2 e^{-x}$

$$y' = (2x)(e^{-x}) + (x^2)(e^{-x}(-1))$$

$$y' = \frac{2x}{e^x} - \frac{x^2}{e^x} = \frac{2x - x^2}{e^x}$$

Derive: $e^{x+y} - xy^2 = \ln y$

$$e^{x+y}(1+y') - (1\cdot y^2 + x\cdot 2yy') = \frac{y'}{y}$$

$$e^{x+y} + y'e^{x+y} - y^2 - 2xyy' = \frac{y'}{y}$$

$$e^{x+y} - y^2 = \frac{y'}{y} + 2xyy' - y'e^{x+y}$$

$$e^{x+y} - y^2 = y'\left(\frac{1}{y} + 2xy - e^{x+y}\right)$$

$$y' = \frac{e^{x+y} - y^2}{\frac{1}{y} + 2xy - e^{x+y}}$$

$$y' = \frac{e^{x+y} - y^2}{\frac{1 + 2xy^2 - ye^{x+y}}{y}} = \frac{y(e^{x+y} - y^2)}{1 + 2xy^2 - ye^{x+y}}$$

70) Find extrema and point(s) of inflection: $f(x) = xe^{-x}$

$$f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x}(-1) = e^{-x} - xe^{-x} = e^{-x}(1-x)$$

$$f''(x) = -1e^{-x} - (e^{-x} - xe^{-x}) = -2e^{-x} + xe^{-x} = e^{-x}(-2+x)$$

$e^{-x}(1-x) = 0$

$e^{-x} = 0$ None

$1 - x = 0$

$x = 1$

$f(1) = 1e^{-1} = \frac{1}{e}$

$\left(1, \frac{1}{e}\right)$ $\wedge$ Max Rel

$f''(1) = -2e^{-1} + 1e^{-1}$

$\frac{-2}{e} + \frac{1}{e} = -\frac{1}{e}$

$e^{-x}(-2+x)$

$-2 + x = 0$

$x = 2$

$f(2) = 2e^{-2} = \frac{2}{e^2}$

$\left(2, \frac{2}{e^2}\right)$ P of I

2

$-$ | $+$

$f''(3) = +$

$f''(1) = -$

48) Derive: $y = x^{3/2} \log_2 \sqrt{x+1}$ $\quad y = x^{3/2} \cdot \frac{1}{2} \log_2 (x+1)$

$$y' = \frac{1}{2}\left(\frac{3}{2}x^{1/2} \cdot \log_2(x+1) + x^{3/2} \cdot \frac{1}{(x+1)\ln 2}\right)$$

$$y' = \frac{\sqrt{x}}{2}\left(\frac{3\log_2(x+1)}{2} + \frac{x}{(x+1)\ln 2}\right)$$

70) $\int_1^e (6^x - 2^x)\,dx$

$$\left[\frac{6^x}{\ln 6} - \frac{2^x}{\ln 2}\right]_1^e = \left(\frac{6^e}{\ln 6} - \frac{2^e}{\ln 2}\right) - \left(\frac{6^1}{\ln 6} - \frac{2^1}{\ln 2}\right)$$

$$\frac{6^e - 6}{\ln 6} + \frac{2 - 2^e}{\ln 2}$$