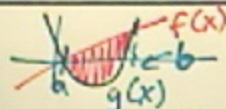


## 7.1 Area of a Region Between Two Curves

$$A = \int_a^b [f(x) - g(x)] dx$$



$$f(x) \geq g(x)$$

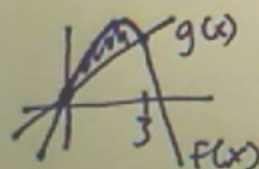
20) Area Between Curves:

$$f(x) = -x^2 + 4x + 1, \quad g(x) = x + 1$$

$$-x^2 + 4x + 1 = x + 1$$

$$0 = x^2 - 3x \quad 0 = x(x-3)$$

$$x = 0 \text{ or } 3$$



$$f(1) = -(1)^2 + 4(1) + 1$$

$$f(1) = 4$$

$$g(1) = 1 + 1 = 2$$

$$\int_0^3 [(-x^2 + 4x + 1) - (x + 1)] dx$$

$$\int_0^3 -x^2 + 3x dx = \left[ -\frac{x^3}{3} + \frac{3x^2}{2} \right]_0^3$$

$$\left( -\frac{27}{3} + \frac{27}{2} \right) - 0 = -9 + \frac{27}{2} = \frac{-18 + 27}{2} = \frac{9}{2}$$

28) Area Between Curves:

$$f(y) = y(2-y), \quad g(y) = -y$$

$$2y - y^2 = -y \quad 0 = y^2 - 3y$$

$$f(1) = 1(2-1) = 1 \quad y = 3 \text{ or } 0$$

$$g(1) = -1$$

$$\int_0^3 (2y - y^2) - (-y) dy$$

$$\int_0^3 -y^2 + 3y dy = \frac{9}{2}$$

36) Area Between Curves:  $y = x^4 - 2x^2$ ,  $y = 2x^2$ 

$$x^4 - 2x^2 = 2x^2 \quad \int_{-2}^0 (2x^2) - (x^4 - 2x^2) dx + \int_0^2 (2x^2) - (x^4 - 2x^2) dx$$

$$x^4 - 4x^2 = 0$$

$$x^2(x^2 - 4) = 0 \quad \int_{-2}^0 -x^4 + 4x^2 dx + \int_0^2 -x^4 + 4x^2 dx$$

$$x = 0, \pm 2$$

$$(-1)^4 - 2(-1)^2 = -1 \quad 2(-1)^2 = 2$$

$$(1)^4 - 2(1)^2 = -1 \quad 2(1)^2 = 2$$

$$\left[ \frac{-x^5}{5} + \frac{4x^3}{3} \right]_{-2}^0 + \left[ \frac{-x^5}{5} + \frac{4x^3}{3} \right]_0^2$$

44) Area Between Curves:

 $f(x) = \sin x$ ,  $g(x) = \cos 2x$ ,  $-\pi/2 \leq x \leq \pi/6$ 

$$\sin x = \cos 2x$$

$$\sin x = -2\sin^2 x \quad 2x - 1 \quad (x+2)(x-1)$$

$$2\sin^2 x + \sin x - 1 = 0 \quad (x+1)(2x-1)$$

$$(\sin x + 1)(2\sin x - 1) = 0$$

$$\sin x = -1 \quad \sin x = \frac{1}{2}$$

$$x = -\frac{\pi}{2} \quad x = \frac{\pi}{6}$$

$$(0) - \left( \frac{+32}{5} - \frac{+32}{3} \right) + \left( \frac{-32}{5} + \frac{32}{3} \right) - (0)$$

$$\frac{-64}{5} + \frac{64}{3}$$

$$\int_{-\pi/2}^{\pi/6} (\cos 2x - \sin x) dx$$

$$f(0) = \sin 0 = 0 \quad \left[ \frac{1}{2} \sin 2x + \cos x \right]_{-\pi/2}^{\pi/6}$$

$$g(0) = \cos(2 \cdot 0) = 1 \quad \left( \frac{1}{2} \sin \frac{\pi}{3} + \cos \frac{\pi}{6} \right) - \left( \frac{1}{2} \sin(-\pi) + \cos(-\pi/2) \right)$$

$$\left( \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) - (0 + 0) = \frac{\sqrt{3}}{4} + \frac{2\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$$

## 7.2 Volume: The Disk Method

Disk:  $V = \pi \int_a^b [R(x)]^2 dx$  ○

12)  $y = 2x^2$ ,  $y = 0$ ,  $x = 2$

a) y-axis  $y = 2(2)^2 = 8$

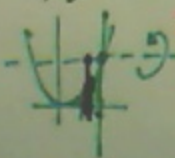


$y = 2x^2$   
 $\frac{y}{2} = x^2$   
 $x = \pm \sqrt{\frac{y}{2}}$

$\pi \int_0^8 (2)^2 - (f(y))^2 dy$

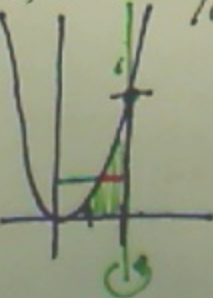
$\pi \int_0^8 4 - \left(\frac{y}{2}\right) dy$

c)  $y = 8$



$\pi \int_0^2 (8)^2 - (8 - 2x^2)^2 dx$

d)  $x = 2$



$\pi \int_0^8 (2 - \sqrt{\frac{y}{2}})^2 dy$

Washer:  $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$   
 larger smaller

b) x-axis  $\pi \int_0^2 (2x^2)^2 dx$

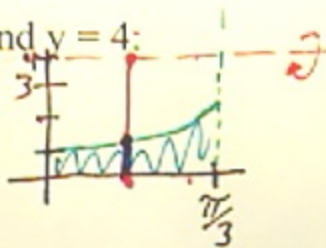


$\pi \int_0^2 4x^4 dx$

$\pi \left[ \frac{4x^5}{5} \right]_0^2$

18) Volume by revolving around  $y = 4$ :

$$y = \sec x, \quad y = 0, \quad 0 \leq x \leq \frac{\pi}{3}$$



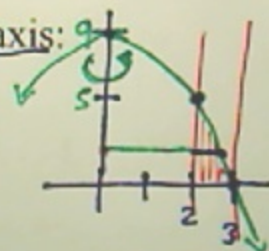
$$\pi \int_0^{\frac{\pi}{3}} (4)^2 - (4 - \sec x)^2 dx$$

$$\pi \int_0^{\frac{\pi}{3}} 16 - (16 - 8\sec x + \sec^2 x) dx$$

$$\pi \int_0^{\frac{\pi}{3}} 8\sec x - \sec^2 x dx = \underline{\hspace{2cm}} u^3$$

32) Volume by revolving around  $y$ -axis:

$$y = 9 - x^2, \quad y = 0, \quad x = 2, \quad x = 3$$



$$V = \pi \int_0^5 (\sqrt{9-y})^2 - (2)^2 dy$$

outer                      inner

$$y = 9 - x^2 \quad V = \pi \int_0^5 (9-y) - (4) dy = \pi \int_0^5 5-y dy = \pi \left[ 5y - \frac{y^2}{2} \right]_0^5$$

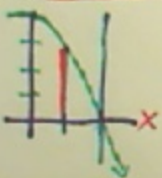
$$\sqrt{x^2} = \sqrt{9-y}$$

$$x = \pm \sqrt{9-y} \quad \rightarrow \pi \left[ 25 - \frac{25}{2} \right] - (0) = \pi \left( \frac{25}{2} \right) \quad V = \left( \frac{25\pi}{2} \right)$$

1) Volume by creating cross sections perpendicular to the x-axis:

$$y = 4 - x^2, y = 0, x = 0, x = 2$$

a) Squares:



$$\int_0^2 (4 - x^2)^2 dx$$

$$\begin{array}{|c|} \hline (4 - x^2) \\ \hline (4 - x^2) \\ \hline \end{array}$$

b) Rectangles: (h=3)

$$\int_0^2 3(4 - x^2) dx$$

$$\begin{array}{|c|} \hline 3 \\ \hline 4 - x^2 \\ \hline \end{array}$$

2) Volume by creating cross sections perpendicular to the y-axis:

$$y = 4 - x^2, y = 0, x = 0, x = 2$$

a) Semicircles:

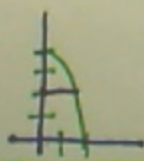
$$r = \frac{\sqrt{4-y}}{2}$$

$$y = 4 - x^2$$

$$x = \sqrt{y+4}$$

$$A = \pi \left( \frac{\sqrt{4-y}}{2} \right)^2$$

$$\frac{1}{8} \int_0^4 \pi(4-y) dy$$



b) Equilateral Triangles:

$$h^2 + \left(\frac{b}{2}\right)^2 = b^2$$

$$h = \sqrt{b^2 - \frac{b^2}{4}} \quad A = \frac{b}{2} \left( \sqrt{\frac{3b^2}{4}} \right)$$

$$\int_0^4 \frac{b}{2} \left( \sqrt{\frac{3b^2}{4}} \right) db \quad b = \sqrt{4-y}$$

